# Unit 5 Review



**Standard Form:**

$y = ax^2 + bx + c$

**Vertex Form:**

$y = a(x-h)^2 + k$

**Intercept Form:**

$y = a(x-p)(x-q)$

## Transformations

| Transformation | Action | Example |
|---|---|---|
| Vertical Shift up | graph moves up | $y = x^2 + 1$ |
| Vertical Shift down | graph moves down | $y = x^2 - 1$ |
| Horizontal Shift left | graph moves left | $y = (x + 1)^2$ |
| Horizontal Shift right | graph moves right | $y = (x - 1)^2$ |
| Vertical stretch | pulls points away from x-axis | $y = 2x^2$ *changes by a factor of 2 |
| Vertical shrink | pushes points toward x-axis | $y = \frac{1}{2}x^2$ *changes by a factor of 1/2 |
| Horizontal shrink | pushes points toward y-axis | $y = (2x)^2$ *changes by a factor of 1/2 |
| Horizontal stretch | pulls points away from y-axis | $y = (\frac{1}{2}x)^2$ *changes by a factor of 2 |
| Reflection across x-axis | graph opens down | $y = -x^2$ |
| Reflection across y-axis | graph opens up | $y = (-x)^2$ |

## Transformations

Describe how the parent function was transformed to create the graph of the function below.

1) $y = -2(x+3)^2 - 1$

reflect over x-axis
vertical stretch by 2
left 3
down 1

2) $y = \left(-\frac{1}{6}x\right)^2$

reflect over y-axis
horizontal stretch by 6

Write the equation of the quadratic that was created after the parent function underwent the transformations listed below.

3) vertical shrink by 2/3
horizontal shift right 7

$y = \frac{2}{3}(x-7)^2$

4) vertical shift down 8
horizontal shrink by 1/5

$y = (5x)^2 - 8$

## Finding VERTEX, AOS, EXTREMA, Y-INTERCEPT

$y = -x^2 + 4x + 1$

$y = -0^2 + 4(0) + 1$

y-int: $(0,1)$

$-\frac{b}{2a} = \frac{-4}{2(-1)} = \frac{-4}{-2} = 2$

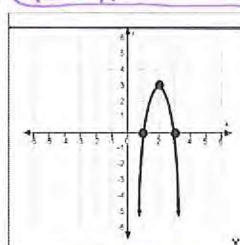
$y = -(2)^2 + 4(2) + 1 = 5$

V: $(2,5)$ ext: max at 5
AOS: $x=2$

$y = 2(x-1)^2 + 3$

V: $(1,3)$
AOS: $x=1$
ext: min at 3
y-int: $(0,5)$

$= 2(0-1)^2 + 3$
$= 2(-1)^2 + 3$
$= 2(1) + 3$
$= 2 + 3$
$= 5$



V: $(2,3)$
AOS: $x=2$
ext: max at 3
y-int: CANNOT TELL

$y = (x+6)(x-1)$

zeros: $-6, 1$

AOS: $x = -2.5$
vertex: $(-2.5, -12.25)$
ext: min at $-12.25$
y-int: $(0,-6)$

$= (-2.5+6)(-2.5-1)$
$= (3.5)(-3.5)$
$= -12.25$

$= (0+6)(0-1)$
$= (6)(-1)$
$= -6$

## Finding AVERAGE RATE OF CHANGE

$y = -x^2 + 4x + 1$

$-2 \le x \le 0$

$a = -2$
$b = 0$
$f(a) = -(-2)^2 + 4(-2) + 1 = -11$
$f(b) = -0^2 + 4(0) + 1 = 1$

$\frac{1-(-11)}{0-(-2)} = \frac{12}{2} = 6$

$y = 2(x-1)^2 + 3$

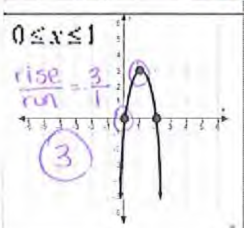
$1 \le x \le 4$

$a = 1$
$b = 4$
$f(a) = 2(1-1)^2 + 3 = 3$
$f(b) = 2(4-1)^2 + 3 = 21$

$\frac{21-3}{4-1} = \frac{18}{3}$

6

$0 \le x \le 1$



$\frac{rise}{run} = \frac{3}{1}$

3

$y = (x+6)(x-1)$

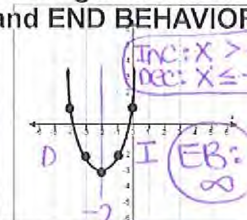
$-2 \le x \le 1$

$a = -2$
$b = 1$
$f(a) = (-2+6)(-2-1) = -12$
$f(b) = (1+6)(1-1) = 0$

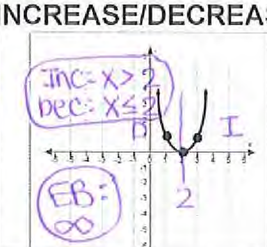
$\frac{0-(-12)}{1-(-2)} = \frac{12}{3}$

4

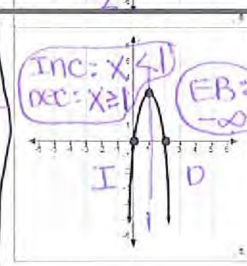
## Finding INTERVAL OF INCREASE/DECREASE and END BEHAVIOR



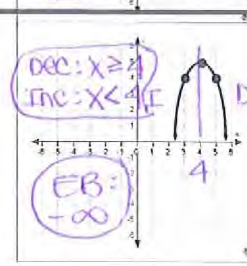
Inc: $x > -2$
Dec: $x \le -2$
EB: $\infty$



Inc: $x > 2$
Dec: $x \le 2$
EB: $\infty$



Inc: $x < 1$
Dec: $x \ge 1$
EB: $-\infty$



Dec: $x \ge 4$
Inc: $x < 4$
EB: $-\infty$

## Finding X-INTERCEPTS and SOLUTIONS

*Remember: solutions, zeros and roots are all the same thing!!*

**USE FACTORING**

$y = 2x^2 + 3x + 1$

$2 \cdot 1 = 2$
$2 + 1 = 3$

$= 2x^2 + 2x + x + 1$
$= (x+1)(2x+1)$

| | $x$ | $1$ |
|---|---|---|
| $2x$ | $2x^2$ | $2x$ |
| $1$ | $x$ | $1$ |

$x+1=0$ $\quad$ $2x+1=0$

sol $x=-1$ $\quad$ $x=-\frac{1}{2}$

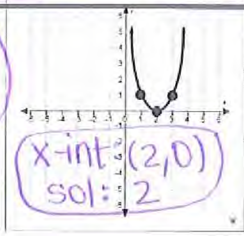
x-int: $(-1,0)$, $(-\frac{1}{2},0)$

**USE TAKING A SQUARE ROOT**

$y = 8x^2 - 200$

$0 = 8x^2 - 200$
$200 = 8x^2$
$25 = x^2$ ← sol
$\pm 5 = x$

x-int $(5,0)$ $(-5,0)$




**USE THE QUADRATIC FORMULA**

$16x^2 - 20x = 4x - 9$

$16x^2 - 20x - 4x + 9 = 0$
$16x^2 - 24x + 9 = 0$

$$\frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)}$$

$$\frac{24 \pm \sqrt{576 - 576}}{32} = \frac{24 \pm \sqrt{0}}{32} = \frac{24 \pm 0}{32}$$

$\frac{24}{32} = \frac{3}{4}$ ← sol $\quad$ x-int: $(3/4, 0)$

## Convert to STANDARD FORM

$y = 2(x+4)(x-3)$

$y = 2(x^2 - 3x + 4x - 12)$
$= 2(x^2 + x - 12)$
$y = 2x^2 + 2x - 24$

$y = -(x-2)^2 - 3$

$y = -(x-2)(x-2) - 3$
$= -(x^2 - 2x - 2x + 4) - 3$
$= -(x^2 - 4x + 4) - 3$
$= -x^2 + 4x - 4 - 3$
$y = -x^2 + 4x - 7$

## Convert to INTERCEPT FORM by FACTORING

$y = x^2 + 10x + 21$

$7 \cdot 3 = 21$ $\quad$ $x^2 + 7x + 3x + 21$
$7 + 3 = 10$

| | $x$ | $7$ |
|---|---|---|
| $x$ | $x^2$ | $7x$ |
| $3$ | $3x$ | $21$ |

$y = (x+7)(x+3)$

$y = 5x^2 + 9x + 4$

$5 \cdot 4 = 20$ $\quad$ $5x^2 + 5x + 4x + 4$
$5 + 4 = 9$

| | $x$ | $1$ |
|---|---|---|
| $5x$ | $5x^2$ | $5x$ |
| $4$ | $4x$ | $4$ |

$y = (5x+4)(x+1)$

## Convert to VERTEX FORM

$y = x^2 + 10x + 21$

$\frac{-10}{2(1)} = \frac{-10}{2} = -5$

$= (-5)^2 + 10(-5) + 21$
$= -4$

$a = 1,\ h = -5,\ k = -4$

$y = (x+5)^2 - 4$

$y = 4x^2 + 8x + 16$

$\frac{-8}{2(4)} = \frac{-8}{8} = -1$

$y = 4(-1)^2 + 8(-1) + 16$
$= 12$

$a = 4,\ h = -1,\ k = 12$

$y = 4(x+1)^2 + 12$

## Application Problems

If a toy rocket is launched vertically upward from ground level with an initial velocity of 128 feet per second, then its height $h$ after $t$ seconds is given by the equation $h(t) = -16t^2 + 128t$ (if air resistance is neglected).

a) How long will it take for the rocket to return to the ground? 8 sec

b) After how many seconds will the rocket be 112 feet above the ground? 1 sec

c) How long will it take the rocket to hit its maximum height?

$-b/2a = -128/2(-16) = -128/-32 = 4$ sec

d) What is the maximum height?

$= -16(4)^2 + 128(4) = 256$ ft

a) $0 = -16t^2 + 128t$

$$\frac{-128 \pm \sqrt{128^2 - 4(-16)(0)}}{2(-16)}$$

$$\frac{-128 \pm \sqrt{16384}}{-32} = \frac{-128 \pm 128}{-32}$$

$\frac{-128 + 128}{-32} = 0$

$\frac{-128 - 128}{-32} = 8$

b) $112 = -16t^2 + 128t$

$0 = -16t^2 + 128t - 112$

$$\frac{-128 \pm \sqrt{128^2 - 4(-16)(-112)}}{2(-16)}$$

$$\frac{-128 \pm \sqrt{16384 - 7168}}{-32} = \frac{-128 \pm \sqrt{9216}}{-32} = \frac{-128 \pm 96}{-32}$$

$\frac{-128+96}{-32} = 1$ $\quad$ $\frac{-128-96}{-32} = 7$

## Identifying Characteristics using a Table

| x | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| y | 15 | 8 | 3 | 0 | -1 | 0 | 3 |

x-intercepts: (2,0), (4,0)
y-intercept: (0,8)
zeros: 2, 4

vertex: (3,-1)
extrema: min at -1
AOS: x = 3

ROC, $3 \leq x \leq 5$: 2

$\frac{3-(-1)}{5-3} = \frac{4}{2} = 2$