



Material covered on Thursday's benchmark:

- Ch 13 (Sections 1, 2, 3, 5): Rational Functions and Equations
- Section 13.4: Rational Functions
- Exponential Functions & Previous Functions Knowledge

1) Joanna's pay for working overtime, p , varies jointly as the number of hours she works, n , and her hourly pay rate, r , and $p = \$103.44$ when $n = 8$ hours and $r = \$8.62$. Find n when $p = \$213.75$ and $r = \$9.50$.

$$n = 15 \text{ hours}$$

2) The cost of packing boxes, c , varies inversely with the number of boxes, b , purchased. If $c = \$0.75$ when $b = 20$, determine the cost of packing 5 boxes.

$$c = \$3.00$$

3) The time required to process a shipment of goods at Wal-Mart varies directly with the number of items in the shipment and inversely with the number of workers assigned. If 15,000 items can be processed by 8 workers in 10 hours, then how long would it take 12 workers to process 20,000 items?

$$t = 8.9 \text{ hours}$$

Simplify the following rational expressions.

$$4) \frac{x+1}{x^2-7x-18} \div \frac{7x^2}{7x^3+14x^2}$$

$$\frac{x+1}{x-9}$$

$$5) \frac{3x^2+18x}{x^2+x-30}$$

$$\frac{3x}{x-5}$$

$$6) \frac{x^2+6x+5}{6x+6} \cdot \frac{x-6}{5x^2+25x} \cdot \frac{15x-30}{3}$$

$$\frac{(x-6)(x-2)}{6x}$$

$$7) \frac{x}{x+3} + \frac{2x+6}{x^2+6x+9}$$

$$\frac{x+2}{x+3}$$

$$8) \frac{2x^2+64}{x^2-64} - \frac{x-4}{x+8}$$

$$\frac{x+4}{x-8}$$

$$9) \frac{2}{y+3} - \frac{y}{y-1} + \frac{y^2+2}{y^2+2y-3}$$

$$\frac{-y}{(y+3)(y-1)}$$

$$10) \frac{\frac{x}{x+2}}{2x+\frac{x}{5}}$$

$$\frac{5}{11(x+2)}$$

$$11) \frac{\frac{x^2+8x+15}{x^2+x-6}}{\frac{x^2+2x-15}{x^2-2x-3}}$$

$$\frac{x+1}{x-2}$$

$$12) \frac{20x^4}{x^2y} \cdot \frac{x^3}{6y^3} \div \frac{10x^2y}{12y^6}$$

$$4x^3y$$

$$13) \frac{x^2+13x+42}{x^3-2x^2-63x}$$

$$\frac{x+6}{x(x-9)}$$

$$14) \frac{\frac{1}{4} + \frac{1}{x}}{\frac{1}{4} - \frac{1}{x^2}}$$

$$\frac{2x}{x-2}$$

$$15) \frac{x}{6x+24} - \frac{4}{x^2+2x-8}$$

$$\frac{x-6}{6(x-2)}$$

Solve and check for extraneous solutions.

$$16) x - \frac{6}{x} = 5$$

$$x = -1, 6$$

$$17) \frac{2}{d+2} + \frac{8}{d-2} = \frac{14}{d^2-4}$$

$$d = \frac{1}{5}$$

$$18) \frac{4}{x^2-4} = \frac{1}{x-2}$$

No solution

Find the characteristics listed of the rational function.

$$19) f(x) = -\frac{2}{x+5} - 1$$

Vertical Asymptote: $x = -5$

Horizontal Asymptote: $y = -1$

Domain: $(-\infty, -5) \cup (-5, \infty)$

Range: $(-\infty, -1) \cup (-1, \infty)$

21) Write a rational function with a hole located at $x = -3$, vertical asymptote of $y = 3$.

$$f(x) = \frac{3x^2 + 7x - 6}{x^2 - x - 12}$$

22) Write a rational function with a hole located at $x = 0$, vertical asymptote of $y = 0$.

$$f(x) = \frac{x}{x}$$

Find the characteristics

$$23) f(x) = \frac{2x^2}{x^2-4}$$

VA
HA
SA
Z
H

Find the characteristics listed of the rational function. List the transformations.

$$19) f(x) = -\frac{2}{x+5} - 1$$

Vertical Asymptote: $x = -5$

Horizontal Asymptote: $y = -1$

Domain: $(-\infty, -5) \cup (-5, \infty)$

Range: $(-\infty, -1) \cup (-1, \infty)$

$$20) f(x) = \frac{1}{x-3} + 2$$

Vertical Asymptote: $x = 3$

Horizontal Asymptote: $y = 2$

Domain: $(-\infty, 3) \cup (3, \infty)$

Range: $(-\infty, 2) \cup (2, \infty)$

21) Write a rational function with a hole located at $x = -3$, vertical asymptotes at $x = 4$, zero of $x = \frac{2}{3}$, and horizontal asymptote of $y = 3$.

$$f(x) = \frac{3x^2 + 7x - 6}{x^2 - x - 12}$$

22) Write a rational function with a hole located at $x = 0$, vertical asymptotes at $x = -9$ and $\frac{1}{6}$ and horizontal asymptote of $y = 0$.

$$f(x) = \frac{x}{6x^3 + 53x^2 - 9x}$$

Find the characteristics of each rational function.

$$23) f(x) = \frac{2x^2 - 5x - 3}{x^2 + 4x - 21}$$

$$24) f(x) = \frac{x^2 + 6x + 5}{x - 2}$$

$$25) f(x) = \frac{x}{x^2 - 9}$$

VA $x = -7$

HA $y = 2$

SA None

Zeros $x = -\frac{1}{2}$

Holes $x = 3$

VA $x = 2$

HA None

SA $y = x + 8$

Zeros $-5, -1$

Holes None

VA $x = \pm 3$

HA $y = 0$

SA None

Zeros 0

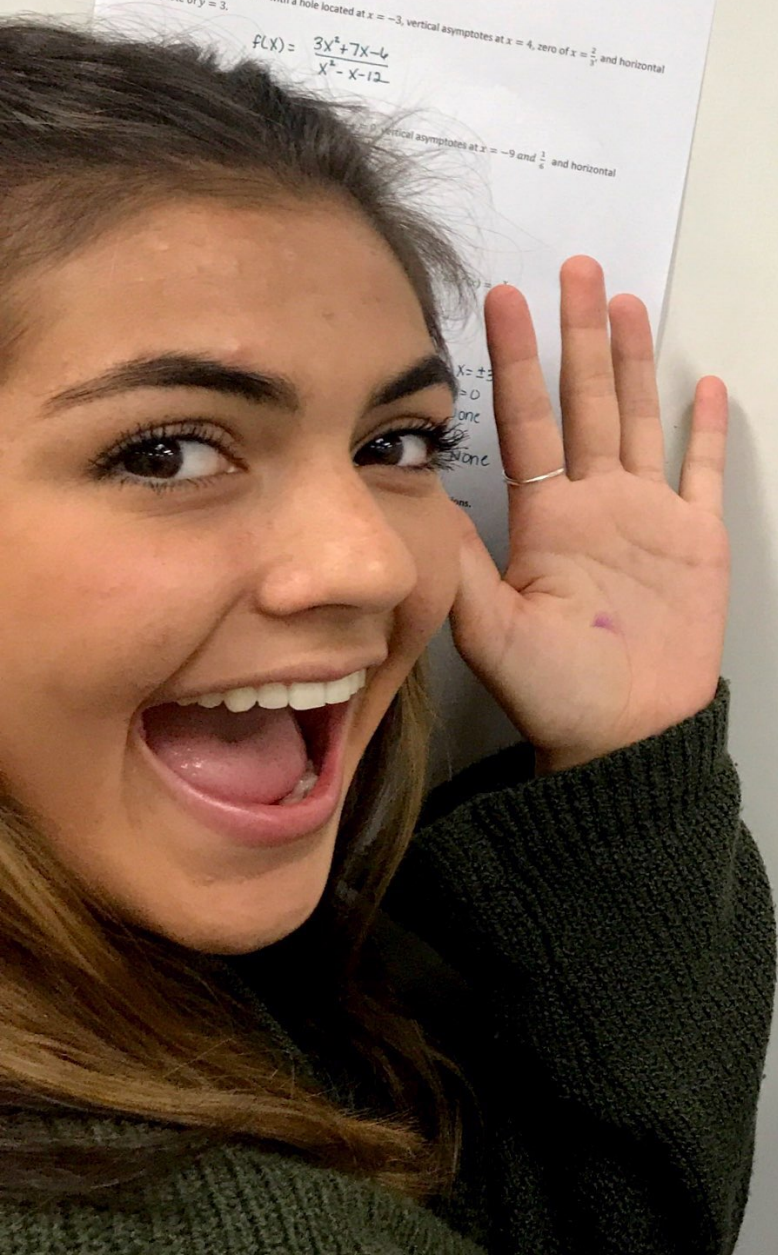
Holes None

Write Question #23 in the translated form of $f(x) = \frac{a}{x-h} + k$ and List All Transformations.

23

$$f(x) = \frac{-13}{x+7} + 2$$

- reflection x-axis
- y stretch by 13
- left 7
- up 2



Find the characteristics listed of the rational function and describe the transformations.

19) $f(x) = -\frac{2}{x+5} - 1$
Vertical Asymptote: $x = -5$
Horizontal Asymptote: $y = -1$
Domain: $(-\infty, -5) \cup (-5, \infty)$
Range: $(-\infty, -1) \cup (-1, \infty)$

20) $f(x) = \frac{1}{x-3} + 2$
Vertical Asymptote: $x = 3$
Horizontal Asymptote: $y = 2$
Domain: $(-\infty, 3) \cup (3, \infty)$
Range: $(-\infty, 2) \cup (2, \infty)$

21) Write a rational function with a hole located at $x = -3$, vertical asymptotes at $x = 4$, zero of $x = \frac{2}{3}$, and horizontal asymptote of $y = 3$.

$f(x) = \frac{3x^2 + 7x - 4}{x^2 - x - 12}$

Vertical asymptotes at $x = -9$ and $\frac{1}{6}$ and horizontal asymptote of $y = 3$.

Given the following exponential functions. Give the domain/range (interval notation), the asymptote equation, end behavior (using limits), the parent function, growth/decay, and transformations.

26) $f(x) = 2 \cdot 4^{x-7} - 6$
D: $(-\infty, \infty)$ R: $(-6, \infty)$
HA: $y = -6$ $f(x) = 4^x$ growth
 $\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -6$
• stretch by 2, right 7, down 6

27) $g(x) = -\left(\frac{1}{2}\right)^{3x+5}$
D: $(-\infty, \infty)$ R: $(-\infty, 0)$
HA: $y = 0$ $g(x) = \left(\frac{1}{2}\right)^x$ decay
 $\lim_{x \rightarrow \infty} f(x) = -\infty$ $\lim_{x \rightarrow -\infty} f(x) = 0$
• ref X-axis, H. shrink $\frac{1}{2}$, left $\frac{5}{3}$

28) $f(x) = \frac{5}{4} \cdot \left(\frac{2}{3}\right)^{\frac{1}{5}x} + 3$
D: $(-\infty, \infty)$ R: $(3, \infty)$
HA: $y = 3$ $f(x) = \left(\frac{2}{3}\right)^x$ decay
 $\lim_{x \rightarrow \infty} f(x) = 3$ $\lim_{x \rightarrow -\infty} f(x) = \infty$
• stretch $\frac{5}{4}$, ref. y-axis, h stretch $\frac{1}{5}$, up 3

29) $g(x) = -5^x - 8$
D: $(-\infty, \infty)$ R: $(-\infty, -8)$
HA: $y = -8$ $g(x) = 5^x$ growth
 $\lim_{x \rightarrow \infty} f(x) = -\infty$ $\lim_{x \rightarrow -\infty} f(x) = -8$
• reflect X-axis
• down 8

For the following functions, give domain/range, equations of any asymptotes, and end behavior (using limits)

30) $f(x) = -x^2 + 6x^2 - 3$
D: $(-\infty, \infty)$ R: $(-\infty, \infty)$
No asymptotes
 $\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$

31) $g(x) = -\sqrt{x-3} + 4$
D: $[3, \infty)$ R: $(-\infty, 4]$
No asymptotes
 $\lim_{x \rightarrow \infty} f(x) = \text{DNE}$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$

32) $f(x) = -4|3x| + 4$
D: $(-\infty, \infty)$ R: $(-\infty, 4]$
No asymptotes
 $\lim_{x \rightarrow \infty} f(x) = -\infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$

33) $g(x) = \frac{2}{x} + 5$
D: $(-\infty, 0) \cup (0, \infty)$
R: $(-\infty, 5) \cup (5, \infty)$
HA: $y = 5$ VA: $x = 0$
 $\lim_{x \rightarrow \infty} f(x) = 5$ $\lim_{x \rightarrow -\infty} f(x) = 5$